

9) INTEGRATION OF TRIG PRODUCTS [MERIT]

Reminder: Product to Sum formulae

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

Examples: Integrate these functions

1) $\int \sin 4x \cos 3x \, dx$

$$\int \sin 4x \cos 3x \, dx$$

$$= \int \frac{1}{2} \sin(4x + 3x) + \frac{1}{2} \sin(4x - 3x) \, dx$$

$$= \int \frac{1}{2} \sin 7x + \frac{1}{2} \sin x \, dx$$

$$= \frac{-1}{2} \cdot \frac{1}{7} \cos 7x - \frac{1}{2} \cos x + c$$

$$= \frac{-1}{14} \cos 7x - \frac{1}{2} \cos x + c$$

2) $\int \cos 6x \cos 7x \, dx$

$$\int \cos 6x \cos 7x \, dx$$

$$= \int \frac{1}{2} \cos(6x + 7x) + \frac{1}{2} \cos(6x - 7x) \, dx$$

$$= \int \frac{1}{2} \cos 13x + \frac{1}{2} \cos(-x) \, dx$$

$$= \int \frac{1}{2} \cos 13x + \frac{1}{2} \cos(x) \, dx$$

remember $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$

$$= \frac{1}{2} \cdot \frac{1}{13} \sin 13x + \frac{1}{2} \sin x + c$$

$$= \frac{1}{26} \sin 13x + \frac{1}{2} \sin x + c$$

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